Reshape the perfect electrical conductor cylinder at will

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Abstract

A general method is proposed to design the cylindrical cloak, concentrator and superscatterer with arbitrary cross section. The method is demonstrated by the design of a perfect electrical conductor (PEC) reshaper which is able to reshape a PEC cylinder arbitrarily by combining the concept of cloak, concentrator and superscatterer together. Numerical simulations are performed to demonstrate its properties.

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The pioneer work on transformation optics and cloaking [1-4] has suggested a new kind of methodology to manipulate the electromagnetic (EM) waves by means of the metamaterials. Based on the transformation optics, various exciting functional devices have been designed both in theory [5-15] and in experiment [16]. Chen *et al.* has recently proposed the "imperfect cloak" [7] which can reduce the scattering cross section of an object. Start from this imperfect cloak, they also suggested a design of "dispersive cloak". In contrast to the imperfect cloak, a concept of "superscatterer" [12] was also proposed which can obtain a giant scattering cross section beyond the device itself. If the inner core of the concentrator [11] is replaced with a PEC cylinder, the concentrator can employed as a device to change the scattering cross section of the PEC cylinder. And its functionality is between the imperfect cloak and the superscatterer. In addition, a cylindrical cloak of arbitrary cross section [10] has been given, which can be easily extended to the imperfect cloak, concentrator, and superscatterer. In this paper, we will show the method of such an extension. And based on our methodology, we will propose a kind of transformation media, which we shall call the "PEC reshaper". An explicit design will be given. The PEC reshaper can reshape the PEC cylinder at will. For example, the effective PEC cylinder can be partially inside the device and partially outside the device. We will demonstrate the properties of the PEC reshaper by using the finite-element methods.

The general coordinate transformation is given by the following relation,

$$\begin{cases} r'(r,\theta) = \frac{b-a}{b-c}(r-b) + b = \frac{b-a}{b-c}r + \frac{a-c}{b-c}b, \\ \theta' = \theta,\ 0 \le \theta < 2\pi \\ z' = z,\ z \in \mathrm{R} \end{cases}, \tag{1}$$

where $a = \rho_1(\theta), b = \rho_2(\theta), c = \rho_3(\theta)$, which are three functions that specify the inner and outer boundaries of the transformation media and the boundary of an imaginary cylinder which will be useful later, respectively. We note that all the boundaries are smooth and non-convex. This transformation maps the field in the domain $\rho_3(\theta) < r < \rho_2(\theta)$ onto another domain $\rho_1(\theta) < r' < \rho_2(\theta)$. From the transformation optics, one can obtain the parameters inside the transformation media in $\rho_1(\theta) < r' < \rho_2(\theta)$. We consider the transverse electric (TE) mode for simplicity and suppose that the inner cylinder ($r' \le \rho_1(\theta)$) is a PEC cylinder.

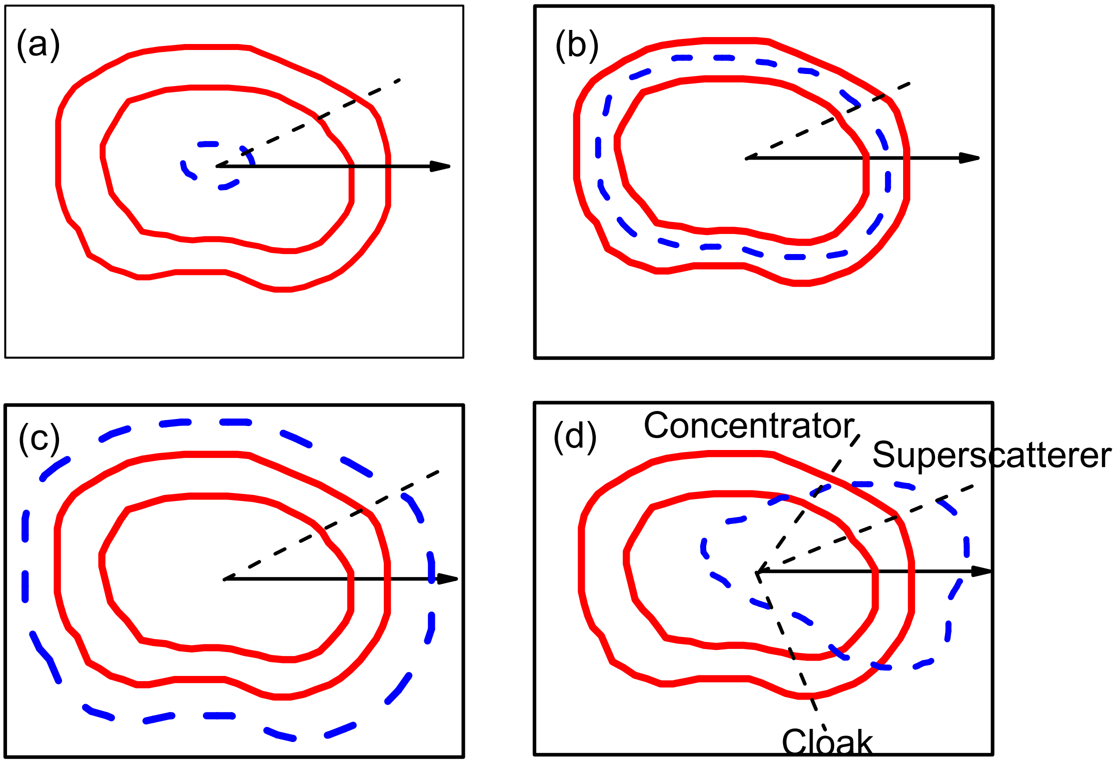

FIG. 1: (Color online) The schematic plot for the coordinate transformation of (a) an imperfect cylindrical cloak with arbitrary cross section, (b) a cylindrical concentrator with arbitrary cross section, (c) a cylindrical superscatterer with arbitrary cross section, and (d) a cylindrical PEC reshaper. The red solid lines denote the outer and inner boundaries of the above devices, while the blue dashed lines outline the effective PEC boundaries in the view of the outside world.

In the view of transformation optics, after coated with the transformation media in $\rho_1(\theta) < r' < \rho_2(\theta)$, the inner PEC cylinder ($r' \leq \rho_1(\theta)$) looks like another effective PEC cylinder in the domain $r \leq \rho_3(\theta)$. If $\rho_3(\theta) \leq \rho_1(\theta)$, the transformation media is an imperfect cylindrical cloak [7, 8] with arbitrary cross section, which can reduce the cross section of the inner PEC cylinder (See in Fig. 1(a)). If $\rho_3(\theta) \equiv 0$, the imperfect cylindrical cloak becomes perfect [1, 10]. If $\rho_1(\theta) \leq \rho_3(\theta) \leq \rho_2(\theta)$, the transformation media is the outer shell of a cylindrical concentrator [11] with arbitrary cross section, which can enhance the cross section of the

inner PEC cylinder (See in Fig. 1(b)). We shall still call the device "concentrator" for simplicity. However, the effective PEC cylinder is still within the domain $r' \leq \rho_2(\theta)$. With the help of the concept of "superscatterer" [12], we can reshape the inner PEC cylinder very versatilely. In the case of superscatterer (i.e. $\rho_2(\theta) \leq \rho_3(\theta)$), the transformation maps the field in the domain $\rho_2(\theta) < r < \rho_3(\theta)$ onto another domain $\rho_1(\theta) < r' < \rho_2(\theta)$ through the mirror of $r' = r = \rho_2(\theta)$. The superscatterer can enhance the cross section of the inner PEC cylinder to that of an effective PEC cylinder which can be much larger than the device (See in Fig. 1(c)). Now we consider an extraordinary case shown in Fig. 1(d), where the effective PEC cylinder is partially inside the inner cylinder, partially within the domain of transformation media and partially outside the device for different angles. Such a device combines cloak, concentrator and superscatterer all together into one case, which we shall call the "PEC reshaper" as it could reshape the PEC boundary more or less arbitrarily.

Before demonstrating the properties of the PEC reshaper, we have to work out the required parameters for it from the transformation optics. The Jacobian is

$$J = \begin{bmatrix} \frac{\partial r'}{\partial r} & \frac{\partial r'}{r\partial\theta} & 0 \\ \frac{r'\partial\theta'}{\partial r} & \frac{r'\partial\theta'}{r\partial\theta} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{2}$$

where $\frac{\partial r'}{\partial r} = \frac{b-a}{b-c}$, $\frac{r'\partial\theta'}{\partial r} = 0$, $\frac{r'\partial\theta'}{r\partial\theta} = \frac{r'}{r}$, and

$$\frac{\partial r'}{r\partial\theta} = \frac{a'(c-b) + b'(a-c) + c'(b-a)}{r(b-c)} \frac{r'-b}{b-a} + \frac{a-c}{b-c} \frac{b'}{r}, \tag{3}$$

with $\rho' = \frac{\partial\rho}{\partial\theta}, \rho = a, b, c, \quad r = \frac{b-c}{b-a} r' + \frac{c-a}{b-a} b$, and $\theta = \theta'$.

The corresponding permittivity and permeability tensors of the transformation media on the circular cylindrical coordinate are [13],

$$\overleftrightarrow{\varepsilon}_c = \overleftrightarrow{\mu}_c = JJ^T / \det(J) = \begin{bmatrix} \mu_{rr} & \mu_{r\theta} & 0 \\ \mu_{\theta r} & \mu_{\theta\theta} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \tag{4}$$

where $\mu_{rr} = \frac{(\frac{\partial r'}{\partial r})^2 + (\frac{\partial r'}{r\partial\theta})^2}{\frac{\partial r'}{\partial r}\frac{r'}{r}}$, $\mu_{r\theta} = \mu_{\theta r} = \frac{\frac{\partial r'}{r\partial\theta}}{\frac{\partial r'}{\partial r}}$, $\mu_{\theta\theta} = \frac{\frac{r'}{r}}{\frac{\partial r'}{\partial r}}$, $\varepsilon_{zz} = \frac{1}{\frac{\partial r'}{\partial r}\frac{r'}{r}}$. We note that the permittivity and permeability tensors here should be written as the functions of positions

in the form of (r', θ').

Consider the TE mode, we have, $\varepsilon_{zz} = \frac{1}{\frac{\partial r'}{\partial r}\frac{r'}{r}}$, and

$$\begin{bmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{xy} & \mu_{yy} \end{bmatrix} = \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix} \begin{bmatrix} \mu_{rr} & \mu_{r\theta} \\ \mu_{r\theta} & \mu_{\theta\theta} \end{bmatrix} \begin{bmatrix} \cos\theta' & \sin\theta' \\ -\sin\theta' & \cos\theta' \end{bmatrix}$$
$$= \begin{bmatrix} \mu_{rr}\cos^2\theta' - 2\mu_{r\theta}\sin\theta'\cos\theta' + \mu_{\theta\theta}\sin^2\theta' & (\mu_{rr} - \mu_{\theta\theta})\sin\theta'\cos\theta' + \mu_{r\theta}(\cos^2\theta' - \sin^2\theta') \\ (\mu_{rr} - \mu_{\theta\theta})\sin\theta'\cos\theta' + \mu_{r\theta}(\cos^2\theta' - \sin^2\theta') & \mu_{rr}\sin^2\theta' + 2\mu_{r\theta}\sin\theta'\cos\theta' + \mu_{\theta\theta}\cos^2\theta' \end{bmatrix}, \tag{5}$$

on Cartesian coordinate [14]. We note that the above parameters are also valid for cloak, concentrator and superscatterer with arbitrary cross section.

With the explicit form of the required parameters, the properties of the PEC reshaper can be demonstrated by numerical simulations with the COMSOL Multiphysics finite element-based electromagnetics solver. Let $a = \rho_1(\theta) = 1m, b = \rho_2(\theta) = 2m$, and $c = \rho_3(\theta) = x_0\cos\theta + \sqrt{a^2 + x_0^2\cos^2\theta}$, where $x_0 = \frac{b^2 - a^2}{2a}$. We choose $c = \rho_3(\theta)$ as a circle for simplicity. A plane wave is normal incident from left to right with unit amplitude and a frequency of 0.1 GHz. Fig. 2(a) and (b) show the snapshots of the total electric and scattering field caused by a PEC cylinder with its outer boundary depicted by $c = \rho_3(\theta)$, respectively. Fig. 2(c) and (d) show the total electric and scattering field induced by the PEC reshaper which is a concentric cylindrical shell. Comparing the similar far-field pattern from Fig. 2(a) and (c), or Fig. 2(b) and (d), we can conclude that the PEC reshaper reshapes the PEC cylinder with $a = \rho_1(\theta) = 1m$ to an effective PEC cylinder with $c = \rho_3(\theta)$. The large overvalued fields in Fig. 2(c) and (d) are caused by the surface mode resonances excited at $c = \rho_3(\theta)$, which are replaced with white flecks [12].

In conclusion, we have proposed the concept of PEC reshaper which combines the concept of cloak, concentrator and superscatterer all together into one case. The PEC reshaper can reshape the PEC boundary arbitrarily if the objective effective PEC cylinder shares a domain with the original PEC cylinder (i.e. there exists $\theta' = \theta \in [0,\ 2\pi)$). The properties of the PEC reshaper are demonstrated by numerical simulations from finite-element methods. The design can be extended to three dimensions straitforwardly.

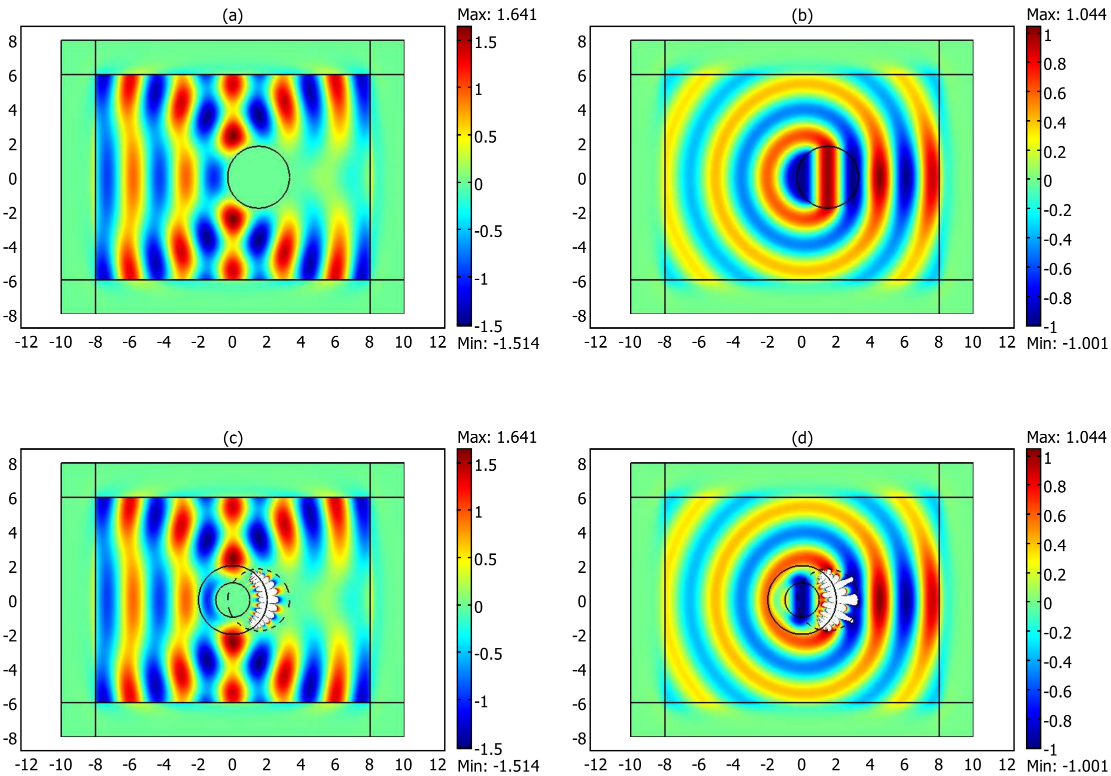

FIG. 2: (Color online) Snapshot of the total and scattering electric field. (a)-(b) The total and scattering electric field caused by a PEC cylinder with its outer boundary depicted by $c = \rho_3(\theta)$, respectively. (c)-(d) The total and scattering electric field induced by the designed PEC reshaper, respectively. The dashed lines in (c) and (d) outline the boundary of the effective PEC cylinder, which is the same to the outer boundary of the PEC cylinder in (a) and (b).

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